

Magnetic brane solutions of Lovelock gravity with nonlinear electrodynamics

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In this paper, we consider logarithmic and exponential forms of nonlinear electrodynamics as a source and obtain magnetic brane solutions of the Lovelock gravity. Although these solutions have no curvature singularity and no horizon, they have a conic singularity with a deficit angle. We investigate the effects of nonlinear electrodynamics and the Lovelock gravity on the value of deficit angle and find that various terms of Lovelock gravity do not affect deficit angle. Next, we generalize our solutions to spinning cases with maximum rotating parameters in arbitrary dimensions and calculate the conserved quantities of the solutions. Finally, we consider nonlinear electrodynamics as a correction of the Maxwell theory and investigate the properties of the solutions.

I. INTRODUCTION

Nonsingular solutions are playing an increasingly important role in physics. The cosmological singularity at the early universe corresponds to an infinite energy density state and therefore, it may probably be essential to consider the quantum gravity to understand the initial state of the Universe. Hence from the cosmological point of view, nonsingular models of the Universe have special position for scientists [1]. From gravitational viewpoint, various regular solutions, such as gravitational instantons, solitons and horizonless magnetic branes (string) solutions, have become the subject of interest in recent years [2–11].

On the other hand, considering four/higher dimensional spacetimes, the cosmic strings/branes are topological defects which are inevitably formed during phase transitions in the early universe [12]. Investigation of the horizonless magnetic solutions and their relations to the topological defects helps us to think about the origin of cosmic magnetic fields [13, 14]. Besides, from geometric point of view, these structures are fascinating objects, in which have no curvature singularity, no horizon, but they have a conic singularity. One of the important motivations for investigating the horizonless magnetic strings/branes comes from the fact that these kinds of solutions may be interpreted as cosmic strings/branes. The horizonless solutions of Einstein and higher derivative gravity theories in the (absence) presence of the Maxwell and dilaton fields have been studied in literature [5, 6]. An extension to include the nonlinear electrodynamics has also been done [7–11].

The purpose of the present paper is constructing a new class of static and spinning magnetic brane solutions which produces a longitudinal magnetic field in the background of anti-de Sitter spacetime. These solutions are the generalization of the solutions of Ref. [11] to higher dimensions and higher derivative gravity.

Derivation of various theories by physicists is for the reason of better description of phenomena in our Universe. It has been confirmed that most of phenomena in the nature are inherently chaotic and may be described with nonlinear theories. In electrodynamics domain, although the Maxwell theory is in agreement with experimental results, it fails regarding some important issues such as self energy of point-like charges which motivates us to regard nonlinear electrodynamics (NED). NED theories may be created from various viewpoint and motivations. For more explanations of some motivations, we refer the reader to the following brief examples; solving the problem of point-like charge self energy, compatible with AdS/CFT correspondence and string theory frames, understanding the nature of different complex systems, obtaining more information and insight regarding to quantum gravity, description of pair creation for Hawking radiation and the behavior of the compact astrophysical objects such as neutron stars and pulsars [15–17]. These are some evidences that motivate one to consider NED theories.

Through last decades different classes of the nonlinear theories were introduced [18–26]. Among the NED theories, the so-called Born-Infeld (BI) type theories are quite special, whose Lagrangians may be originated from the string theory. It has been shown that the low-energy limit of heterotic string theory in electrodynamics side leads to a quartic correction of the Maxwell field strength [27]. Moreover, one finds that all order loop corrections may be summed up as a BI type Lagrangian [28–30]. Recently, it has been considered two kinds of BI type Lagrangians to examine the possibility of black hole solutions [20–25]. Although there are some analogues between the BI type theories, one can find that there exist some differences between them.

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In recent years a renewed interest has grown in higher dimensional spacetime as well as higher dimensional gravity [31]. The main reason comes from the fact that these theories emerge in the effective low-energy action of string theory in gravitational side [32–35]. One of the special classes of higher derivative modifications of Einstein (EN) gravity is the Lovelock theory [36] which is a ghost free model [37, 38]. Regarding the postulates of general relativity, most of physicists believe that the Lovelock Lagrangian is a natural generalization of the EN gravity to higher dimensions. Besides, Lovelock gravity may solve some of problems of the Einstein theory such as the normalization problem, and hence it is a well-defined model [39–41]. In this paper, we consider the Lovelock gravity in presence of two classes of BI type NED models and obtain their horizonless solutions. We also investigate the effect of NED as a correction to the Maxwell theory.

The layout of this paper will be this: First we introduce the suitable field equations regarding to the Lovelock gravity coupled with different magnetic sources that we are interested in. Next, we obtain static solutions for metric function. Then, we will consider spinning magnetic string and by employing counterterm method, we calculate conserved quantities. Last section will be devoted to closing remarks.

II. STATIC SOLUTIONS

Recently, Dias and Lemos [4] have introduced an interesting spacetime with magnetic brane interpretation which is horizonless. The mentioned metric in d -dimensions may be written as

$$ds^2 = -\frac{\rho^2}{l^2}dt^2 + \frac{d\rho^2}{f(\rho)} + l^2 f(\rho) d\phi^2 + \frac{\rho^2}{l^2} dX^2, \quad (1)$$

where $dX^2 = \sum_{i=1}^{d_3} dx_i^2$ is the Euclidean metric on the d_3 -dimensional submanifold (hereafter we denote $(d-i)$ with d_i). The angular coordinate ϕ is dimensionless and ranges in $[0, 2\pi]$, while x_i range in $(-\infty, \infty)$. This metric provides us horizonless solutions that are of our interest. Now, we are going to obtain the solutions of first, second and third order of the Lovelock gravity in the presence of NED with the following field equations

$$\partial_a (\sqrt{-g} L_F F^{ab}) = 0, \quad (2)$$

$$\Lambda g_{ab} + G_{ab}^{(1)} + \alpha_2 G_{ab}^{(2)} + \alpha_3 G_{ab}^{(3)} = \frac{1}{2} g_{ab} L(F) - 2 L_F F_{ac} F_b^c, \quad (3)$$

where $L_F = \frac{dL(F)}{dF}$, in which $L(F)$ is the Lagrangian of NED, $\Lambda = -\frac{d_1 d_2}{2l^2}$ and $G_{ab}^{(1)} = R_{ab} - \frac{1}{2} g_{ab} R$ are, respectively, the cosmological constant and the Einstein tensor, α_i 's are the Lovelock coefficients and

$$G_{\mu\nu}^{(2)} = 2(R_{\mu\sigma\kappa\tau} R_\nu^{\sigma\kappa\tau} - 2R_{\mu\rho\nu\sigma} R^{\rho\sigma} - 2R_{\mu\sigma} R^\sigma_\nu + R R_{\mu\nu}) - \frac{\mathcal{L}^{(2)}}{2} g_{\mu\nu}, \quad (4)$$

$$\begin{aligned} G_{\mu\nu}^{(3)} = & -3(4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^\lambda_{\nu\tau\mu} - 8R^{\tau\rho}_{\lambda\sigma} R^{\sigma\kappa}_{\tau\mu} R^\lambda_{\nu\rho\kappa} + 2R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\lambda\rho} R^\lambda_{\tau\mu} \\ & - R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\rho} R_{\nu\mu} + 8R^\tau_{\nu\sigma\rho} R^{\sigma\kappa}_{\tau\mu} R^\rho_{\kappa} + 8R^\sigma_{\nu\tau\kappa} R^{\tau\rho}_{\sigma\mu} R^\kappa_{\rho} \\ & + 4R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\mu\rho} R^\rho_{\tau} - 4R_\nu^{\tau\sigma\kappa} R_{\sigma\kappa\tau\rho} R^\rho_{\mu} + 4R^{\tau\rho\sigma\kappa} R_{\sigma\kappa\tau\mu} R_{\nu\rho} + 2R R_\nu^{\kappa\tau\rho} R_{\tau\rho\kappa\mu} \\ & + 8R^\tau_{\nu\mu\rho} R^\rho_{\sigma} R^\sigma_{\tau} - 8R^\sigma_{\nu\tau\rho} R^\tau_{\sigma} R^\rho_{\mu} - 8R^{\tau\rho}_{\sigma\mu} R^\sigma_{\tau} R_{\nu\rho} - 4R R^\tau_{\nu\mu\rho} R^\rho_{\tau} \\ & + 4R^{\tau\rho} R_{\rho\tau} R_{\nu\mu} - 8R^\tau_{\nu} R_{\tau\rho} R^\rho_{\mu} + 4R R_{\nu\rho} R^\rho_{\mu} - R^2 R_{\nu\mu}) - \frac{\mathcal{L}^{(3)}}{2} g_{\mu\nu}, \end{aligned} \quad (5)$$

where $\mathcal{L}^{(2)}$ and $\mathcal{L}^{(3)}$ denote the Lagrangians of the Gauss-Bonnet (GB) and third order the Lovelock (TOL) gravities, given as

$$\mathcal{L}^{(2)} = R_{\mu\nu\gamma\delta} R^{\mu\nu\gamma\delta} - 4R_{\mu\nu} R^{\mu\nu} + R^2, \quad (6)$$

$$\begin{aligned} \mathcal{L}^{(3)} = & 2R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\rho\tau} R^{\rho\tau}_{\mu\nu} + 8R^{\mu\nu}_{\sigma\rho} R^{\sigma\kappa}_{\nu\tau} R^{\rho\tau}_{\mu\kappa} + 24R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\nu\rho} R^\rho_{\mu} \\ & + 3R R^{\mu\nu\sigma\kappa} R_{\sigma\kappa\mu\nu} + 24R^{\mu\nu\sigma\kappa} R_{\sigma\mu} R_{\kappa\nu} + 16R^{\mu\nu} R_{\nu\sigma} R^\sigma_{\mu} - 12R R^{\mu\nu} R_{\mu\nu} + R^3. \end{aligned} \quad (7)$$

In this work, we take into account the recently proposed interesting NED models [21]. One of them is the Soleng model which is logarithmic form and another one has exponential form which was proposed by Hendi with the following explicit forms

$$L(F) = \begin{cases} \beta^2 \left[\exp\left(-\frac{F}{\beta^2}\right) - 1 \right] & \text{ENEF} \\ -8\beta^2 \ln\left(1 + \frac{F}{8\beta^2}\right) & \text{LNEF} \end{cases}, \quad (8)$$

where β is the nonlinearity parameter and the Maxwell invariant is $F = F_{ab}F^{ab}$, in which $F_{ab} = \partial_a A_b - \partial_b A_a$ is the electromagnetic field tensor and A_a is the gauge potential. It is easy to show that the electric field comes from the time component of the vector potential (A_t), while the magnetic field is associated with the angular component (A_ϕ). Since we are looking for the magnetic solutions, we consider the following form of gauge potential

$$A_\mu = h(\rho)\delta_\mu^\phi. \quad (9)$$

Using Eq. (9) with the mentioned NED, one can show that the electromagnetic field equation (2) reduces to the following differential equations

$$\begin{cases} (\rho l^2 \beta^2 - 4\rho h'^2) h'' + d_2 l^2 \beta^2 h' = 0 & \text{ENEF} \\ (4\rho l^2 \beta^2 - r h'^2) h''^2 + 4d_2 h' (l^2 \beta^2 + \frac{1}{4} h'^2) = 0 & \text{LNEF} \end{cases}, \quad (10)$$

where the prime and the double prime denote the first and second derivatives with respect to ρ . Solving these equations one obtains

$$h(\rho) = \begin{cases} \frac{l\beta}{2} \int \sqrt{-L_{W1}} d\rho & \text{ENEF} \\ \frac{\beta^2 \rho^{d_1}}{q d_1} - \frac{\beta^2}{q} \int \Gamma_1 \rho^{d-2} d\rho & \text{LNEF} \end{cases}, \quad (11)$$

where q is an integration constant which is related to the electric charge, $L_{W1} = \text{LambertW}\left(-\left(\frac{4ql}{\beta \rho^{d_2}}\right)^2\right)$ and $\Gamma_1 = \sqrt{1 - \left(\frac{2ql}{\beta \rho^{d_2}}\right)^2}$. Taking into account the mentioned gauge potential, one finds the nonzero components of electromagnetic field are

$$F_{\phi\rho} = -F_{\rho\phi} = \begin{cases} \frac{2ql^2}{\rho^{d_2}} \exp\left(-\frac{L_{W1}}{2}\right), & \text{ENEF} \\ \frac{\beta^2 \rho^{d_2}}{q} (1 - \Gamma_1), & \text{LNEF} \end{cases}. \quad (12)$$

In order to obtain real solutions for the electromagnetic field, we should restrict the coordinate ρ with a lower bound ρ_0 . It means

$$\rho > \rho_0 = \begin{cases} \left(\frac{4ql}{\beta}\right)^{1/d_2} \exp\left(\frac{1}{2d_2}\right), & \text{ENEF} \\ \left(\frac{2ql}{\beta}\right)^{1/d_2}, & \text{LNEF} \end{cases}.$$

We should note that for large values of β all relations reduce to the corresponding relations of the Maxwell theory. Besides, one can find that obtained results of electromagnetic fields reduce to those of Ref. [11] in four dimensions.

In order to obtain the metric function, $f(\rho)$, one can use nonzero components of the gravitational field equation, (3). After cumbersome calculations, we find that there are two different differential equations with the following explicit forms

$$e_t = \mathcal{K}_1 + \alpha_2 \mathcal{K}_2 + \alpha_3 \mathcal{K}_3 = 0, \quad (13)$$

$$e_\rho = \mathcal{K}_{11} + \alpha_2 \mathcal{K}_{22} + \alpha_3 \mathcal{K}_{33} = 0, \quad (14)$$

where

$$\mathcal{K}_1 = -\rho^6 \left(\frac{\rho \mathcal{A}'}{d_2} + \mathcal{A} \right) - \beta^2 \rho^6 \times \begin{cases} 1 - \exp\left(\frac{-2h'^2}{l^2 \beta^2}\right), & \text{ENEF} \\ -8 \ln\left(\frac{4l^2 \beta^2}{4l^2 \beta^2 + h'^2}\right), & \text{LNEF} \end{cases},$$

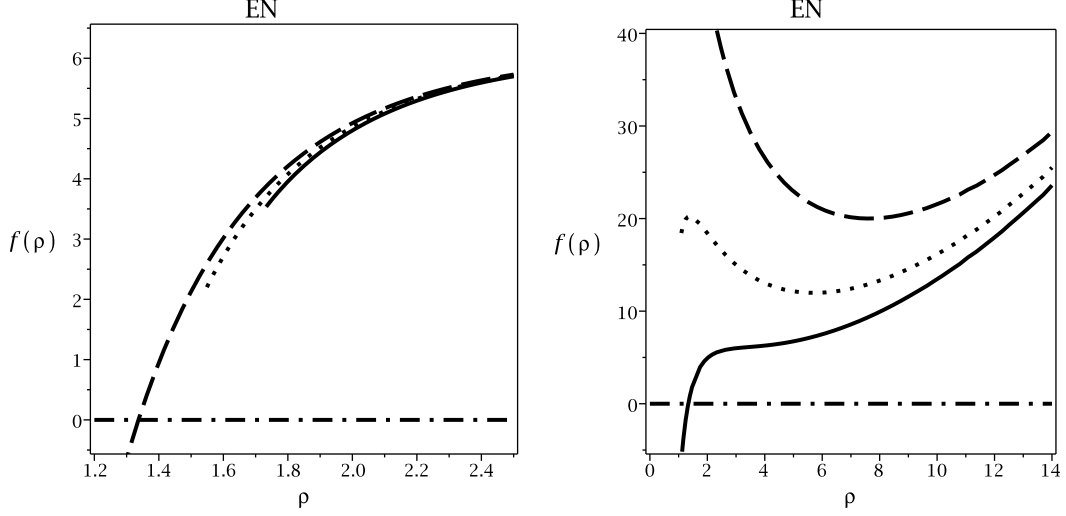


FIG. 1: **LNEF branch of EN gravity:** $f_{EN}(\rho)$ versus ρ for $l = 3$, $q = 1$ and $d = 4$.
 Left panel: $m = 0.5$, $\beta = 2$ (continuous line), $\beta = 2.5$ (dotted line) and $\beta = 5$ (dashed line).
 Right panel: $\beta = 5$, $m = 0.5$ (continuous line), $m = 1$ (dotted line), $m = 2$ (dashed line).

$$\mathcal{K}_2 = d_3 d_4 \rho^4 \left[2f f'' + 2f'^2 + \frac{4d_5 f f'}{\rho} + \frac{d_5 d_6 f^2}{\rho^2} \right],$$

$$\mathcal{K}_3 = -d_3 d_4 d_5 d_6 d_7 d_8 f \rho^2 \left[\frac{3f f'' + 6f'^2}{d_7 d_8} + \frac{6f f'}{d_8 \rho} + \frac{f^2}{\rho^2} \right],$$

$$\mathcal{K}_{11} = \rho^6 \mathcal{A} - \beta^2 \rho^6 \times \begin{cases} \frac{4h'^2}{l^2 \beta^2 \rho} \exp\left(\frac{-2h'^2}{l^2 \beta^2}\right) + \exp\left(\frac{-2h'^2}{l^2 \beta^2}\right) - 1, & \text{ENEF} \\ 8 \left(\frac{2}{1 + \left(\frac{2l\beta}{h'}\right)^2} + \ln \left[1 + \left(\frac{h'}{2l\beta}\right)^2 \right] \right), & \text{LNEF} \end{cases},$$

$$\mathcal{K}_{22} = -d_2 d_3 d_4 d_5 f \rho^2 \left(f + \frac{2\rho f'}{d_5} \right),$$

$$\mathcal{K}_{33} = d_2 d_3 d_4 d_5 d_6 d_7 f^2 \left(f + \frac{3\rho f'}{d_7} \right).$$

Now, we desire to obtain higher dimensional magnetic brane solutions in the EN, GB and TOL gravities, separately. One can set $\alpha_3 = 0$ to obtain the GB solutions and for $\alpha_2 = \alpha_3 = 0$, we obtain magnetic solutions of the EN gravity. After some simplifications, we obtain

$$f_{EN} = \frac{2ml^3}{\rho^{d_3}} - \frac{2\Lambda\rho^2}{d_1 d_2} + \begin{cases} \frac{8\beta^2 \rho^2}{d_1 d_2} + \frac{-8\beta^2 \left(\int \rho^{d_2} \left[\Gamma_1 + \ln \left(\frac{\beta^2 \rho^{2d_2} (1-\Gamma)}{2l^2 d^2} \right) \right] d\rho \right)}{d_2 \rho^{d_3}} & \text{LNEF} \\ -\frac{\beta^2 \rho^2}{d_1 d_2} + \frac{4lq\beta \left(\int \left[\sqrt{-L_{W1}} + \frac{1}{\sqrt{-L_{W1}}} \right] d\rho \right)}{d_2 \rho^{d_3}} & \text{ENEF} \end{cases}, \quad (15)$$

$$f_{GB} = \frac{\rho^2}{2d_3 d_4 \alpha_2} \left(1 - \Psi^{1/2} \right), \quad (16)$$

$$f_{TOL} = \frac{\rho^2}{d_3 d_4 \alpha_2} \left(1 - \Psi^{1/3} \right), \quad (17)$$

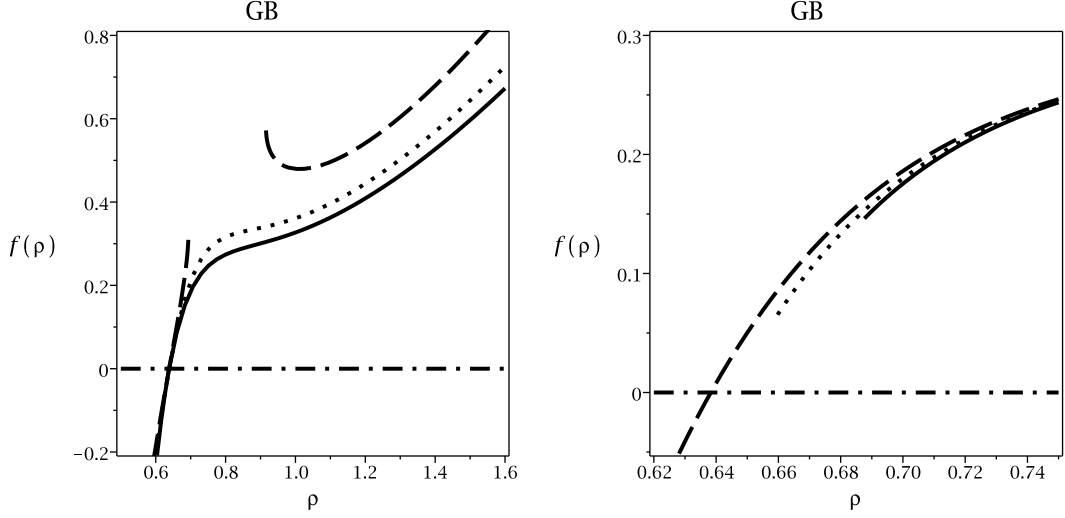


FIG. 2: **LNEF branch of GB gravity:** $f_{GB}(\rho)$ versus ρ for $l = 2$, $q = 0.1$, $m = 0.005$ and $d = 7$. Left panel: $\beta = 10$, $\alpha_2 = 0.01$ (continuous line), $\alpha_2 = 0.03$ (dotted line) and $\alpha_2 = 0.06$ (dashed line). Right panel: $\alpha_2 = 0.01$, $\beta = 2.6$ (continuous line), $\beta = 3.2$ (dotted line), $\beta = 10$ (dashed line).

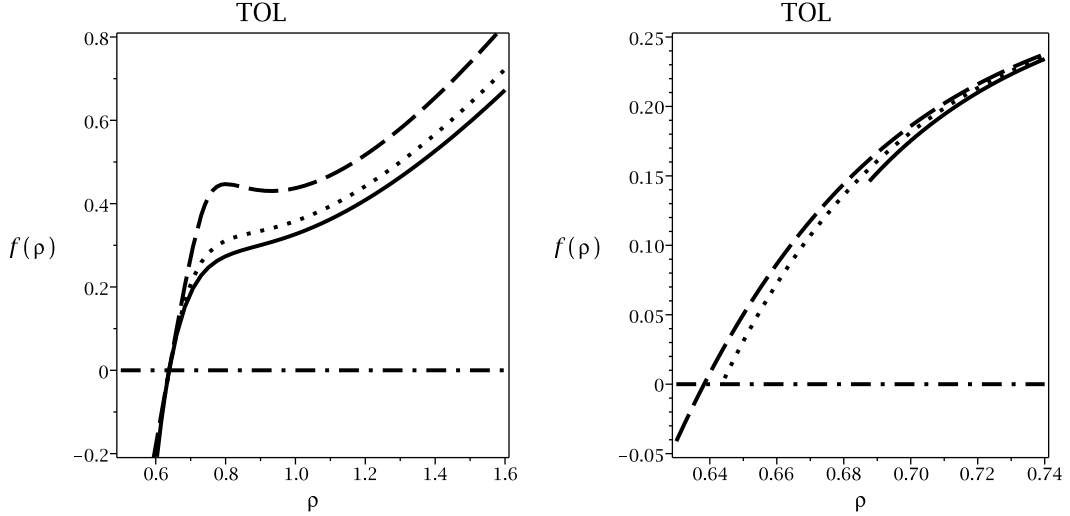


FIG. 3: **LNEF branch of TOL gravity:** $f_{TOL}(\rho)$ versus ρ for $l = 2$, $q = 0.1$, $m = 0.005$ and $d = 7$. Left panel: $\beta = 10$, $\alpha_2 = 0.01$ (continuous line), $\alpha_2 = 0.03$ (dotted line) and $\alpha_2 = 0.06$ (dashed line). Right panel: $\alpha_2 = 0.01$, $\beta = 2.6$ (continuous line), $\beta = 3.2$ (dotted line), $\beta = 10$ (dashed line).

where

$$\Psi = 1 + \frac{2\chi d_3 d_4 \alpha_2}{d_1 d_2} \left(\Lambda - \frac{d_1 d_2 l^3 m}{\rho^{d_1}} + \mathcal{W} \right), \quad (18)$$

with

$$\mathcal{W} = \begin{cases} 4\beta^2 \left\{ \ln \left(\frac{\beta^2 \rho^{2d_2}}{2l^2 q^2} [1 - \Gamma_1] \right) - \frac{(2d_2+1)\Gamma_1}{d_1} \right\} + \frac{16d_3^2 l^2 q^2}{\rho^{2d_2} d_1 d_3} \mathcal{F} & \text{LNEF} \\ \beta^2 \left[\frac{1}{2} + \frac{2d_1 q l}{\beta \rho^{d_1}} \int \left(\sqrt{-L_{W1}} + \frac{1}{\sqrt{-L_{W1}}} \right) d\rho \right] & \text{ENEf} \end{cases}, \quad (19)$$

in which $\chi = 4$ and 3 for the GB theory and the TOL gravity, respectively, \mathcal{F} is ${}_2F_1 \left(\left[\frac{1}{2}, \frac{d_3}{2d_2} \right], \left[\frac{3d_2-1}{2d_2} \right], \frac{4l^2 q^2}{\beta^2 \rho^{2d_2}} \right)$, m is an integration constant related to total finite mass of the solutions and we set $\alpha_3 = \frac{d_3 d_4}{3d_5 d_6} \alpha_2^2$ for more simplifications of TOL gravity solutions.

A. Properties of solutions:

At the first step, we are going to discuss the geometric properties of the solutions. To do this, we look for possible black hole solutions with obtaining the curvature singularities and their horizons. We usually calculate the Kretschmann scalar, $R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}$, to achieve essential singularity. Considering the mentioned spacetime, (1), it is easy to show that

$$R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = f''^2 + 2d_2 \left(\frac{f'}{\rho} \right)^2 + 2d_2d_3 \left(\frac{f}{\rho^2} \right)^2. \quad (20)$$

Inserting the metric function, $f(\rho)$, in Eq. (20) and using numerical analysis, one finds that the Kretschmann scalar diverges at $\rho = \rho_0$ and it is finite for $\rho > \rho_0$ and naturally one may think that there is a curvature singularity located at $\rho = \rho_0$. In what follows, we state an important point, in which confirms that the spacetime never achieves $\rho = \rho_0$. As one can confirm, easily, the metric function has positive value for large values of $\rho \gg \rho_0$. So two cases may occur. For the first case, $f(\rho)$ is a positive definite function with no root and therefore, the singularity called as a naked singularity which we are not interested in. We consider the second case, in which the metric function has one or more real positive root(s) larger than ρ_0 .

From Figs. 1 – 3, we find that there is a ρ_{min} ($\rho_{min} = \rho_0$) in which for $\rho \geq \rho_{min}$ the metric function is real. These figures show that increasing the nonlinearity parameter leads to decreasing ρ_{min} . Since we are looking for the metric function with at least one real root, we should adjust the metric parameters with a suitable range of nonlinearity parameter to obtain $f(\rho = \rho_{min}) \leq 0$.

Moreover, Fig. 3 indicates that although metric function of the TOL gravity is real for arbitrary ρ , in GB gravity one encounters with an imaginary interval for some values of the GB parameter. In other words, in GB gravity we should adjust the metric parameters with suitable interval of α to obtain a real metric function with at least one real root. Besides, Fig. 3 shows that the root of metric function does not depend on the Lovelock parameters.

Now, we denote r_+ as the largest real positive root of $f(\rho)$. The metric function is negative for $\rho < r_+$ and positive for $\rho > r_+$ and hence, the metric signature may change from $(- + + + \dots +)$ to $(- - - + \dots +)$ in the range $0 < \rho < r_+$. Taking into account this apparent change of signature of the metric, we conclude that one cannot extend the spacetime to $\rho < r_+$. In order to get rid of this incorrect extension, one may use the following suitable transformation with introducing a new radial coordinate r

$$\begin{aligned} r^2 &= \rho^2 - r_+^2, \\ \rho \geq r_+ &\iff r \geq 0. \end{aligned} \quad (21)$$

Using the mentioned transformation with $d\rho = \frac{r}{\sqrt{r^2 + r_+^2}} dr$ one finds that the metric (1) should change to

$$ds^2 = -\frac{r^2 + r_+^2}{l^2} dt^2 + \frac{r^2}{(r^2 + r_+^2)f(r)} dr^2 + l^2 f(r) d\phi^2 + \frac{r^2 + r_+^2}{l^2} dX^2. \quad (22)$$

It is worthwhile to mention that with this new coordinate, the electromagnetic field and the metric functions lead to the following form

$$F_{r\phi} = \begin{cases} \frac{2ql^2}{(r^2 + r_+^2)^{\frac{d_2}{2}}} \exp\left(-\frac{L_W}{2}\right), & \text{ENEF} \\ \frac{\beta^2(r^2 + r_+^2)^{\frac{d_2}{2}}}{q} (1 - \Gamma), & \text{LNEF} \end{cases}, \quad (23)$$

$$f_{EN} = \frac{2ml^3}{(r^2 + r_+^2)^{\frac{d_3}{2}}} - \frac{2\Lambda(r^2 + r_+^2)}{d_1 d_2} + \begin{cases} \frac{8\beta^2(r^2 + r_+^2)}{d_1 d_2} + \frac{-8\beta^2 \left(\int r(r^2 + r_+^2)^{\frac{d_3}{2}} \left[\Gamma + \ln\left(\frac{\beta^2(r^2 + r_+^2)^{\frac{d_2}{2}}}{2l^2 q^2} (1 - \Gamma) \right) \right] dr \right)}{d_2 (r^2 + r_+^2)^{\frac{d_2 - 3}{2}}} & \text{LNEF} \\ -\frac{\beta^2(r^2 + r_+^2)}{d_1 d_2} + \frac{4lq\beta \left(\int \left(\sqrt{-L_W} + \frac{1}{\sqrt{-L_W}} \right) \frac{r}{\sqrt{r^2 + r_+^2}} dr \right)}{d_2 (r^2 + r_+^2)^{\frac{d_3}{2}}} & \text{ENEF} \end{cases}, \quad (24)$$

$$f_{GB} = \frac{(r^2 + r_+^2)}{2d_3 d_4 \alpha_2} \left(1 - \Psi^{1/2} \right), \quad (25)$$

$$f_{TOL} = \frac{(r^2 + r_+^2)}{d_3 d_4 \alpha_2} (1 - \Psi^{1/3}), \quad (26)$$

where

$$\Psi = 1 + \frac{2\chi d_3 d_4 \alpha_2}{d_1 d_2} \left(\Lambda - \frac{d_1 d_2 l^3 m}{(r^2 + r_+^2)^{d_1/2}} + \mathcal{W}_1 \right), \quad (27)$$

with

$$\mathcal{W}_1 = \begin{cases} 4\beta^2 \left\{ \ln \left(\frac{\beta^2 (r^2 + r_+^2)^{d_2}}{2l^2 q^2} [1 - \Gamma] \right) - \frac{(2d_2+1)\Gamma}{d_1} \right\} + \frac{16d_2^2 l^2 q^2}{(r^2 + r_+^2)^{d_2} d_1 d_3} \mathcal{F} & \text{LNEF} \\ \beta^2 \left[\frac{1}{2} + \frac{2d_1 q l}{\beta (r^2 + r_+^2)^{\frac{d_1}{2}}} \int \left(\sqrt{-L_W} + \frac{1}{\sqrt{-L_W}} \right) \frac{r}{\sqrt{r^2 + r_+^2}} dr \right] & \text{ENEF} \end{cases}, \quad (28)$$

in which $L_W = LambertW \left(-\frac{16q^2 l^2}{\beta^2 (r^2 + r_+^2)^{d_2}} \right)$, $\mathcal{F} = {}_2F_1 \left(\left[\frac{1}{2}, \frac{d_3}{2d_2} \right], \left[\frac{3d_2-1}{2d_2} \right], \frac{4l^2 q^2}{\beta^2 (r^2 + r_+^2)^{d_2}} \right)$ and $\Gamma = \sqrt{1 - \frac{4q^2 l^2}{\beta^2 (r^2 + r_+^2)^{d_2}}}$.

Since we suppose that $r_+ \geq \rho_0$, the solutions (electromagnetic field and metric functions) are real for $r \geq 0$. In addition, the function $f(r)$ given in Eqs. (24)-(26) is positive in the whole spacetime and is zero at $r = 0$.

Although the Kretschmann scalar does not diverge in the range $0 \leq r < \infty$, one can show that there is a conical singularity at $r = 0$. One can investigate the conic geometry by using the *circumference/radius* ratio. Using the Taylor expansion, in the vicinity of $r = 0$, we find

$$f(r) = f(r)|_{r=0} + \left(\frac{df(r)}{dr} \Big|_{r=0} \right) r + \frac{1}{2} \left(\frac{d^2 f(r)}{dr^2} \Big|_{r=0} \right) r^2 + O(r^3) + \dots, \quad (29)$$

where

$$f(r)|_{r=0} = \frac{df(r)}{dr} \Big|_{r=0} = 0,$$

and it is a matter of calculation to show that regardless of gravity branches (EN, GB and TOL), we will have following relation

$$\frac{d^2 f(r)}{dr^2} \Big|_{r=0} = -\frac{2\Lambda}{d_2} + \frac{2}{d_2} E_0 + \frac{2r_+}{d_1 d_2} E'_0 \neq 0, \quad (30)$$

where $E_0 = E(r)|_{r=0}$, in which $E(r)$ denotes the electromagnetic part of metric functions (third term of Eq. (24) and \mathcal{W}_1 in Eq. (27), and $E'_0 = \frac{dE(r)}{dr} \Big|_{r=0}$. With employing obtained results, one can show that

$$\lim_{r \rightarrow 0^+} \frac{1}{r} \sqrt{\frac{g_{\phi\phi}}{g_{rr}}} = \lim_{r \rightarrow 0^+} \frac{\sqrt{r^2 + r_+^2} l f(r)}{r^2} = \frac{l r_+}{2} \frac{d^2 f(r)}{dr^2} \Big|_{r=0} \neq 1, \quad (31)$$

which confirms that as the radius r tends to zero, the limit of the *circumference/radius* ratio is not 2π and therefore the spacetime has a conical singularity at $r = 0$. This canonical singularity may be removed if one identifies the coordinate ϕ with the period

$$\text{Period}_\phi = 2\pi \left(\lim_{r \rightarrow 0} \frac{1}{r} \sqrt{\frac{g_{\phi\phi}}{g_{rr}}} \right)^{-1} = 2\pi (1 - 4\mu), \quad (32)$$

where μ is given by

$$\mu = \frac{1}{4} \left[1 - \frac{2}{l r_+} \left(\frac{d^2 f(r)}{dr^2} \Big|_{r=0} \right)^{-1} \right]. \quad (33)$$

In other words, the near origin limit of the metric (22) describes a locally flat spacetime which has a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$. Using the Vilenkin procedure, one can interpret μ as the mass

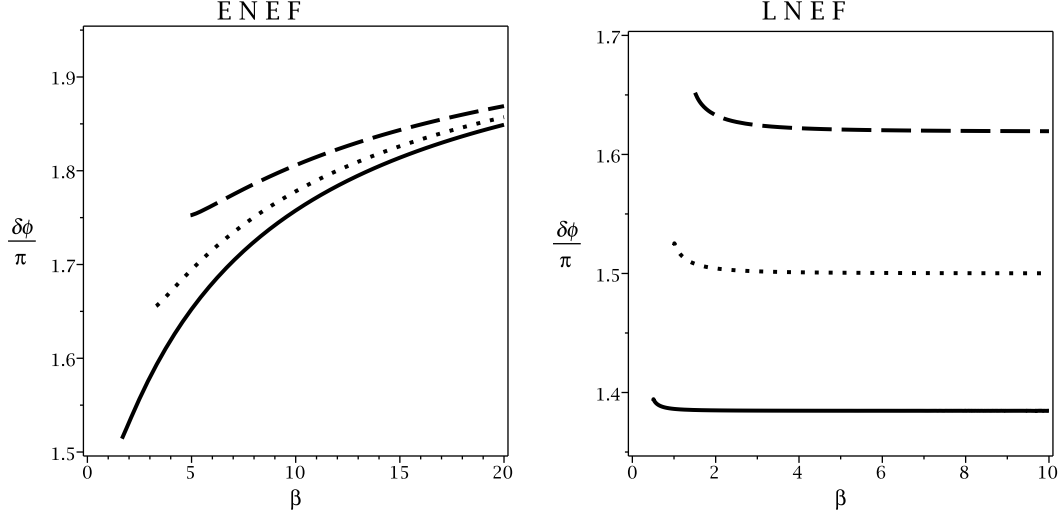


FIG. 4: $\delta\phi/\pi$ versus β for $d = 4$, $l = 1$ and $r_+ = 2$.
 Left panel (ENEf): $q = 1$ (continuous line), $q = 2$ (dotted line) and $q = 3$ (dashed line).
 Right panel (LNEf): $q = 1$ (continuous line), $q = 2$ (dotted line) and $q = 3$ (dashed line).

per unit volume of the magnetic brane [42]. It is evident from (30) and (33) that deficit angle is independent of the Lovelock coefficients and is only a function of cosmological constant and electromagnetic field.

It is obvious that the nonlinearity of electrodynamics can change the value of deficit angle $\delta\phi$. In order to investigate the effects of nonlinearity, r_+ , q and dimensionality, we plot $\delta\phi$ versus β and r_+ (Figs. 4-8). Figs. 4 and 5 show that, for ENEf branch, deficit angle is an increasing function of nonlinearity parameter while for LNEf branch it is a decreasing function of β . In addition, figures of deficit angle show that there is a minimum for nonlinearity parameter β_{\min} in which for $\beta < \beta_{\min}$, the obtained values for deficit angle are not real (see Figs. 4-5). Besides, one finds β_{\min} increases as the charge parameter of magnetic branes increases, whereas for increasing value of r_+ , β_{\min} decreases (see Figs. 4 and 5).

The figures of the deficit angle versus r_+ (see Figs. 6 and 7) show that there is also a minimum $r_{+\min}$ in which for $r_+ \geq r_{+\min}$ the deficit angle is real. For large values of β , the deficit angle is an increasing function of r_+ (see Figs. 6 and 7). These figures show that there is an extremum r_{+ext} that for $r_{+\min} \leq r_+ \leq r_{+ext}$, deficit angle is a decreasing function of r_+ whereas for $r_+ \geq r_{+ext}$ the deficit angle is an increasing function of β (see Figs. 6 and 7).

Considering the fact that obtained results are magnetic branes in arbitrary dimensions, studying the effect of dimensionality on deficit angle is another important issue. Figs 7 and 8 show that for fixed values of metric parameters, the deficit angle is an increasing function of d . Also, as one can see, β_{\min} is a decreasing function of dimensionality and for higher dimensions β_{\min} goes to zero (see Fig. 8). Also, numerical analysis confirm that $r_{+\min}$ is an increasing function of dimensionality (see Fig. 7).

III. A CLASS OF SPINNING SOLUTIONS

In this section, we generalize the static spacetime to the case of rotating solutions. As we know, the rotation group in d -dimensions is $SO(d-1)$ with $[(d-1)/2]$ independent rotation parameters, in which $[x]$ denotes the integer part of x . The rotating magnetic solutions with $k \leq [(d-1)/2]$ rotation parameters may be written as

$$\begin{aligned}
 ds^2 = & -\frac{r^2 + r_+^2}{l^2} \left(\Xi dt - \sum_{i=1}^k a_i d\phi^i \right)^2 + f(r) \left(\sqrt{\Xi^2 - 1} dt - \frac{\Xi}{\sqrt{\Xi^2 - 1}} \sum_{i=1}^k a_i d\phi^i \right)^2 \\
 & + \frac{r^2 dr^2}{(r^2 + r_+^2)f(r)} + \frac{r^2 + r_+^2}{l^2(\Xi^2 - 1)} \sum_{i < j}^k (a_i d\phi_j - a_j d\phi_i)^2 + \frac{r^2 + r_+^2}{l^2} dX^2,
 \end{aligned} \tag{34}$$

where $\Xi = \sqrt{1 + \sum_i^k a_i^2/l^2}$, dX^2 is the Euclidean metric on the $(d-k-2)$ -dimensional submanifold with volume V_{d-k-2} and $f(r)$ is the same as $f(r)$ given in Eqs. (24)-(26) for various gravity. We should note that the non-vanishing

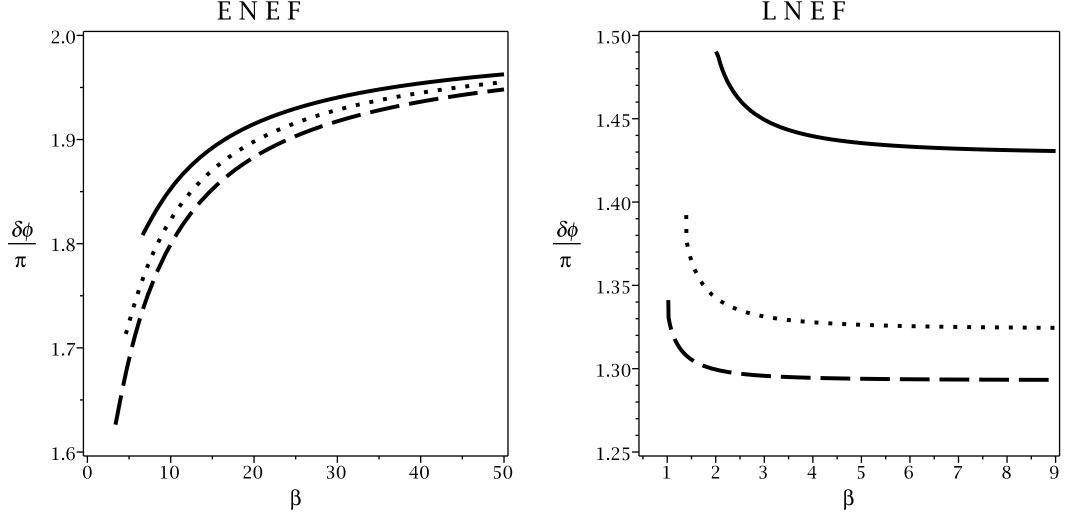


FIG. 5: $\delta\phi/\pi$ versus β for $d = 4$, $l = 1$ and $q = 1$.

Left panel (ENEf): $r_+ = 1$ (continuous line), $r_+ = 1.2$ (dotted line) and $r_+ = 1.4$ (dashed line).

Right panel (LNEf): $r_+ = 1$ (continuous line), $r_+ = 1.2$ (dotted line) and $r_+ = 1.4$ (dashed line).

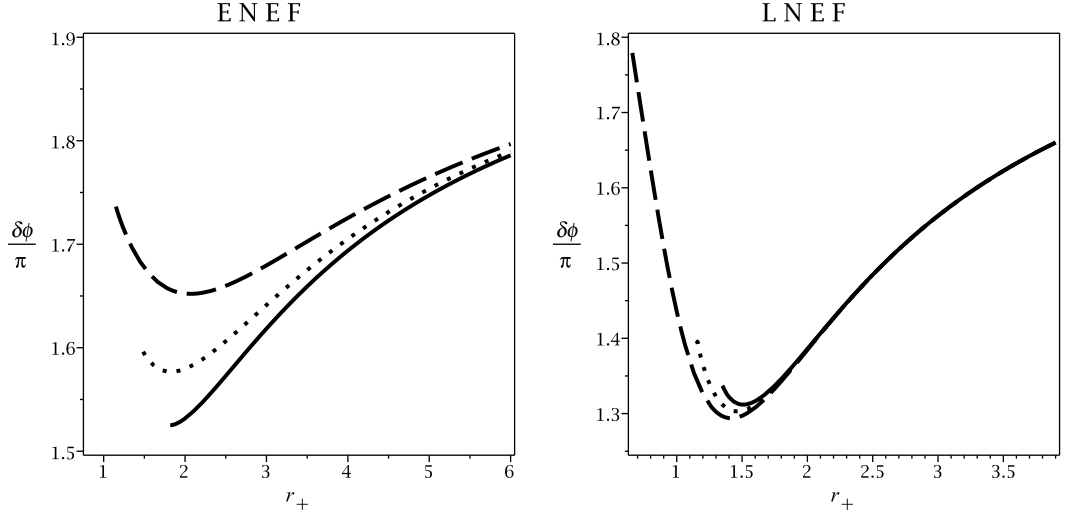


FIG. 6: $\delta\phi/\pi$ versus r_+ for $d = 4$, $l = 2$ and $q = 1$.

Left panel (ENEf): $\beta = 2$ (continuous line), $\beta = 3$ (dotted line) and $\beta = 5$ (dashed line).

Right panel (LNEf): $\beta = 1.1$ (continuous line), $\beta = 1.5$ (dotted line) and $\beta = 5$ (dashed line).

components of electromagnetic field are

$$F_{rt} = -\frac{(\Xi^2 - 1)}{\Xi a_i} F_{r\phi_i} = -\frac{(\Xi^2 - 1)}{\Xi a_i} \times \begin{cases} \frac{2ql^2}{(r^2 + r_+^2)^{d_2/2}} \exp\left(-\frac{L_W}{2}\right), & \text{ENEf} \\ \frac{\beta^2 (r^2 + r_+^2)^{d_2/2}}{q} (1 - \Gamma), & \text{LNEf} \end{cases}. \quad (35)$$

Again, we should note that although this rotating spacetime has no curvature singularity and horizon, it has a conical singularity at $r = 0$.

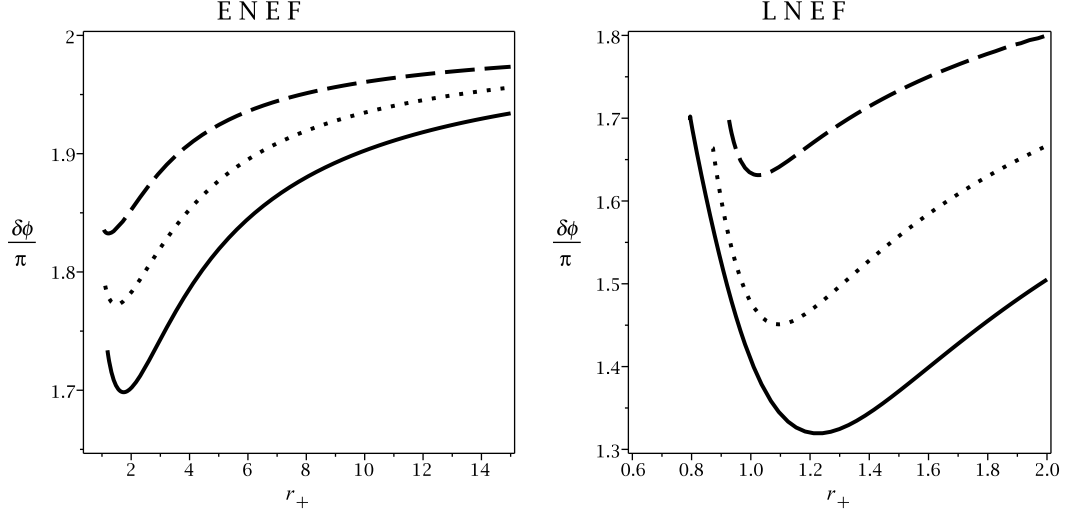


FIG. 7: $\delta\phi/\pi$ versus r_+ for $\beta = 4$, $l = 1$ and $q = 1$.
 Left panel (ENEf): $d = 5$ (continuous line), $d = 7$ (dotted line) and $d = 11$ (dashed line).
 Right panel (LNEf): $d = 5$ (continuous line), $d = 7$ (dotted line) and $d = 11$ (dashed line).

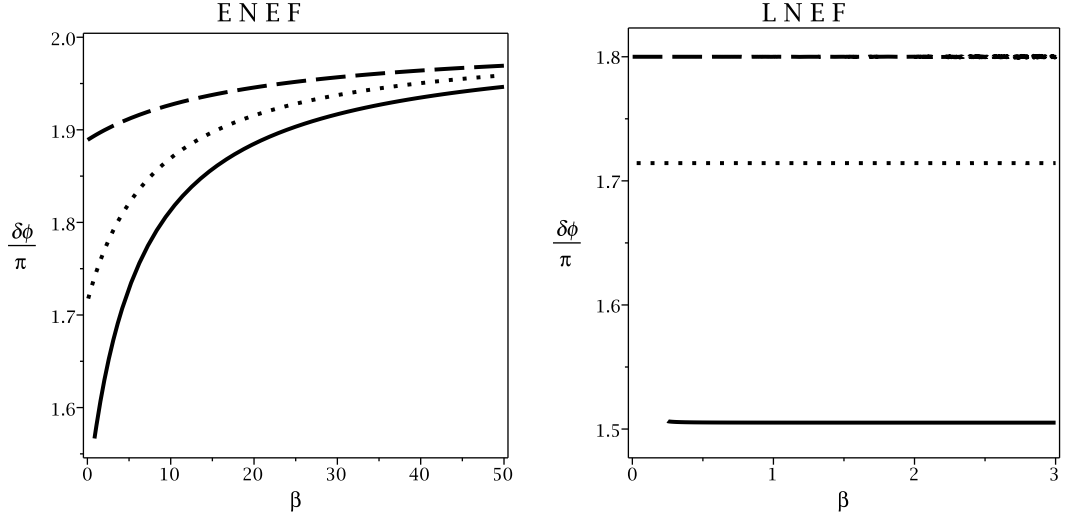


FIG. 8: $\delta\phi/\pi$ versus β for $r_+ = 2$, $l = 1$ and $q = 1$.
 Left panel (ENEf): $d = 5$ (continuous line), $d = 7$ (dotted line) and $d = 11$ (dashed line).
 Right panel (LNEf): $d = 5$ (continuous line), $d = 7$ (dotted line) and $d = 11$ (dashed line).

A. Conserved Quantities

Here, we calculate the angular momentum and mass density of the magnetic solutions. In order to obtain finite conserved quantities for the asymptotically AdS solutions, one may use the counterterm method [43]. Here, for the asymptotically AdS solutions of the Lovelock gravity with flat boundary, $\hat{R}_{abcd}(\gamma) = 0$ (our solutions), the finite energy momentum tensor is [44]

$$T^{ab} = \frac{1}{8\pi} \left[(K^{ab} - K\gamma^{ab}) + 2\alpha_2(3J^{ab} - J\gamma^{ab}) + 3\alpha_3(5P^{ab} - P\gamma^{ab}) + \frac{d_2}{l_{eff}}\gamma^{ab} \right], \quad (36)$$

where l_{eff} is a function of l and α , and when α goes to zero (Einstein solutions), l_{eff} reduces to l . In Eq. (36), K^{ab} is the extrinsic curvature of the boundary, K is its trace, γ^{ab} is the induced metric of the boundary, and J and P are,

respectively, trace of J^{ab} and P^{ab} , where

$$J_{ab} = \frac{1}{3}(K_{cd}K^{cd}K_{ab} + 2KK_{ac}K_b^c - 2K_{ac}K^{cd}K_{db} - K^2K_{ab}), \quad (37)$$

and

$$\begin{aligned} P_{ab} = & \frac{1}{5}\{[K^4 - 6K^2K^{cd}K_{cd} + 8KK_{cd}K_e^dK^{ec} - 6K_{cd}K^{de}K_{ef}K^{fc} + 3(K_{cd}K^{cd})^2]K_{ab} \\ & - (4K^3 - 12KK_{ed}K^{ed} + 8K_{de}K_f^eK^{fd})K_{ac}K_b^c - 24KK_{ac}K^{cd}K_{de}K_b^e \\ & + (12K^2 - 12K_{ef}K^{ef})K_{ac}K^{cd}K_{db} + 24K_{ac}K^{cd}K_{de}K^{ef}K_{bf}\}. \end{aligned} \quad (38)$$

In order to compute the conserved charges, one can write the boundary metric in Arnowitt-Deser-Misner form

$$\gamma^{ab}dx^a dx^b = -N^2 dt^2 + \sigma_{ij}(d\varphi^i + V^i dt)(d\varphi^j + V^j dt), \quad (39)$$

where the coordinates φ^i are the angular variables parameterizing the hypersurface of constant r around the origin and N and V^i are the lapse and shift functions, respectively. The quasilocal conserved quantities associated with the stress tensors of Eq. (36) are

$$\mathcal{Q}(\xi) = \int_{\mathcal{B}} d^{d_2} \varphi \sqrt{\sigma} T_{ab} n^a \xi^b, \quad (40)$$

where σ is the determinant of the metric σ_{ij} , and n^a is the timelike unit normal vector to the boundary \mathcal{B} and ξ is a Killing vector field. The rotating magnetic spacetime (34) has two conserved quantities which are associated with the Killing vectors $\xi = \partial/\partial t$ and $\zeta_i = \partial/\partial \phi^i$. The total mass and angular momentum of the magnetic brane solutions per unit volume V_{d-k-2} , given by

$$M = \int_{\mathcal{B}} d^{d_2} x \sqrt{\sigma} T_{ab} n^a \xi^b = \frac{(2\pi)^k}{4} [d_1(\Xi^2 - 1) + 1] m, \quad (41)$$

$$J_i = \int_{\mathcal{B}} d^{d_2} x \sqrt{\sigma} T_{ab} n^a \zeta_i^b = \frac{(2\pi)^k}{4} \Xi d_1 m a_i, \quad (42)$$

where the mass parameter m comes from the fact that $\lim_{r \rightarrow 0} f(r) = 0$. Our last step will be devoted to calculate the electric charge of the magnetic solutions. To do so, we should consider the projections of the electromagnetic field tensor on a special hypersurface. The electric charge per unit volume V_{d-k-2} can be found by calculating the flux of the electromagnetic field at infinity, yielding

$$Q = \frac{(2\pi)^k}{2} q l \sqrt{\Xi^2 - 1}, \quad (43)$$

which shows that the electric charge is proportional to the magnitude of the rotation parameters and is zero for the static solutions ($\Xi = 1$). This is due to the fact that the electric field, F_{tr} , vanishes for the static solutions. In addition, since the asymptotically behavior of the electromagnetic field is the same as that of the Maxwell theory, the nonlinearity does not affect the total electric charge.

IV. NED AS A CORRECTION

It is arguable that, instead of considering nonlinear theories of the Maxwell field, one can use the method in which the nonlinearity is playing as a correction term. In other words, one is free to consider nonlinearity as a perturbation to linear theory and construct a new nonlinear theory. This treatment is justified with the following reasons. First, in order to find experimental results for a nonlinear electromagnetic fields, one should consider its weak nonlinearity and not strong. This is due to the fact that the Maxwell theory has acceptable consequences in most domains and the perturbed nonlinear theory of electrodynamics may increases the Maxwell accuracy. On the other hand, in order to avoid the complexity of nonlinear theories and obtaining interesting solutions, it is logical to consider the dominant nonlinearity terms and use them in order to study a nonlinear theory. As for BI types of nonlinear electrodynamics for large values of nonlinearity parameter they have same structure with a little differences in some factors. One can

show that the first and second leading order terms are, respectively, the Maxwell Lagrangian and quadratic power of the Maxwell invariant. Therefore, in this section we consider following Lagrangian as a source and study the effects of additional correction to the Maxwell theory (MC) as nonlinear electromagnetic field on solutions:

$$L(F) = -F + \eta F^2 + O(\eta^2). \quad (44)$$

One may follow the procedure of previous sections with the mentioned Lagrangian (44) and the metric (34) to obtain

$$F_{rt} = -\frac{(\Xi^2 - 1)}{\Xi a_i} F_{r\phi_i} = -\frac{(\Xi^2 - 1)}{\Xi a_i} \left(\frac{q}{(r^2 + r_+^2)^{\frac{d_4}{2}}} - \frac{4q^3\eta}{l^2 (r^2 + r_+^2)^{\frac{3d_2}{2}}} \right). \quad (45)$$

Inserting Eq. (45) in the gravitational field equations, we find the following metric functions for the EN, the GB and the TOL gravities in the presence of Lagrangian (44)

$$f_{EN} = \frac{2Ml^3}{(r^2 + r_+^2)^{d_3/2}} - \frac{2(r^2 + r_+^2)}{d_1 d_2} \left(\Lambda - \frac{4d_1 l^2 q^2}{d_3 (r^2 + r_+^2)^{d_2}} + \frac{32d_1 l^4 q^4 \eta}{(3d - 7)(r^2 + r_+^2)^{2d_2}} \right), \quad (46)$$

$$f_{GB} = \frac{(r^2 + r_+^2)}{2d_3 d_4 \alpha_2} \left(1 - \Psi^{\frac{1}{2}} \right), \quad (47)$$

$$f_{TOL} = \frac{(r^2 + r_+^2)}{d_3 d_4 \alpha_2} \left(1 - \Psi^{\frac{1}{3}} \right), \quad (48)$$

where

$$\Psi = 1 + \frac{2\chi d_3 d_4 \alpha_2}{d_1 d_2} \left(\Lambda - \frac{d_1 d_2 l^3 M}{(r^2 + r_+^2)^{d_1/2}} - \frac{4d_1 l^2 q^2}{d_3 (r^2 + r_+^2)^{d_2}} + \frac{32d_1 l^4 q^4 \eta}{(3d_2 - 1)(r^2 + r_+^2)^{2d_2}} \right), \quad (49)$$

and $\chi = 4$ and 3 for the GB theory and the TOL gravity, respectively.

We should note that, regardless of various coefficients, one can obtain these solutions, directly, by suitable series expansions of Eqs. (24) – (26). In addition, in agreement with Eqs. (41), (42) and (43), independent calculations show that the conserved charges do not depend on the nonlinearity parameter of BI type NED theories.

Here, we are in position to study the deficit angle. To do so, we employ the method that was mentioned in previous sections and plot various appropriate graphs. It is a matter of calculation to show that the second order derivation of the metric with respect to radial coordinate will be in the following form for all mentioned gravity branched

$$\left. \frac{d^2 f(r)}{dr^2} \right|_{r=0} = -\frac{2\Lambda}{d_2} - \frac{8l^2 q^2}{d_2 r_+^{2d_2}} + \frac{64l^4 q^4}{d_2 r_+^{4d_2}} \eta + O(\eta^2), \quad (50)$$

where confirms that deficit angle does not depend on the Lovelock coefficient.

Studying the effects of charge parameter show that, for very small values of q and $\eta = 0$, calculated deficit angle is nonzero and is a decreasing function of charge (see left panel in Fig. 9). As charge increases, for certain range of correction parameter, deficit angle is negative and there is a η_0 where calculated deficit angle is zero. This η_0 is an increasing function of charge (see left panel in Fig. 9). As for the effects of r_+ , plotted graphs have similar behavior as charge whereas the effects of r_+ are exactly opposite of the effects of charge (see right panel in Fig. 9).

Considering different values of correction parameter, the deficit angle versus r_+ shows that, calculated deficit angles have different behaviors. For small values of nonlinearity, three different behaviors are seen for different regions of r_+ in which these regions are specified with r_{+Div1} and r_{+Div2} (see left panel in Fig. 10). For $0 < r_+ < r_{+Div1}$, deficit angle is an decreasing function of r_+ and in $r_+ = r_{+Div1}$, there is a divergency. In this region calculated deficit angles are positive and real valued and in case of $r_{+Div1} < r_+ < r_{+Div2}$ for calculated deficit angle, first it is a decreasing and then increasing function of r_+ and for $r_+ = r_{+Div2}$ second divergency happens. Next, for $r_+ > r_{+Div2}$, one finds that deficit angle is an increasing function of r_+ but there exists a region in which calculated deficit angles are negative and for an r_{+0} deficit angle is zero (see left panel in Fig. 10).

Next, for larger values of nonlinearity, there are regions identified with specific values naming r_{+1} , r_{+ext} and r_{+2} . For $0 < r_+ < r_{+1}$, deficit angles for different values of nonlinearity parameter are almost the same. In other words,

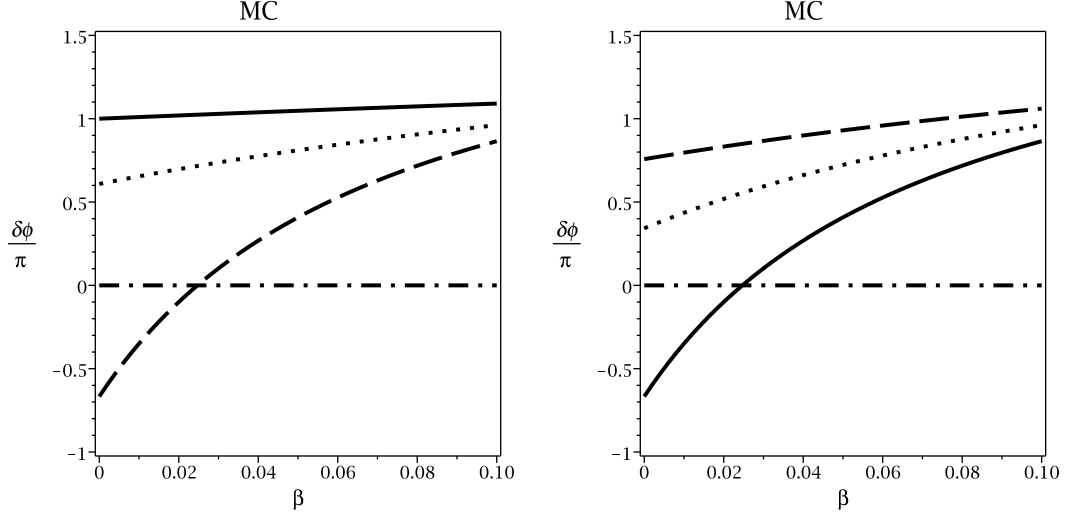


FIG. 9: $\delta\phi/\pi$ versus η for $l = 1$ and $d = 4$.

Left panel: $r_+ = 2$, $q = 2$ (continuous line), $q = 2.5$ (dotted line) and $q = 3$ (dashed line).

Right panel: $q = 3$, $r_+ = 2$ (continuous line), $r_+ = 2.1$ (dotted line) and $r_+ = 2.2$ (dashed line).

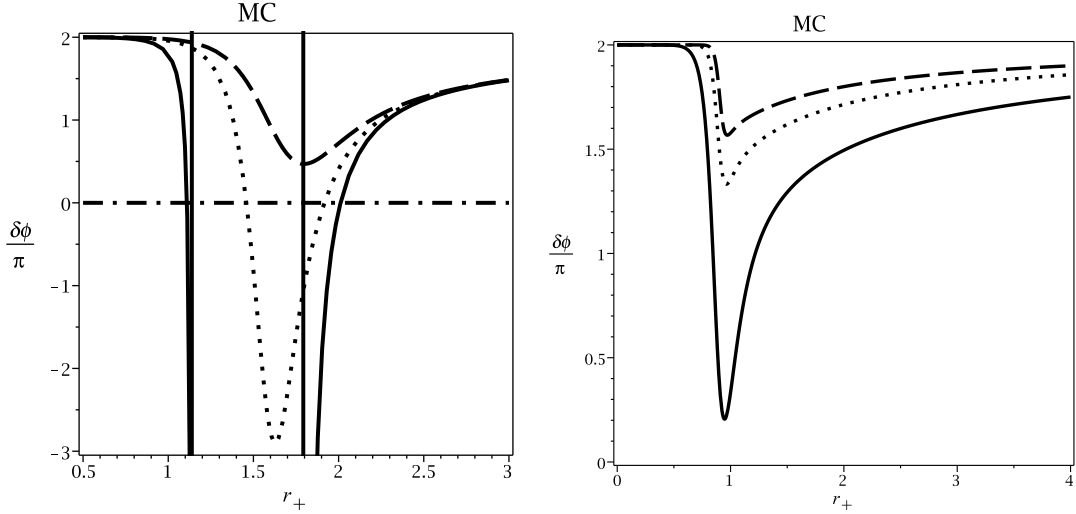


FIG. 10: $\delta\phi/\pi$ versus r_+ for $l = 1$.

Left panel: $q = 3$, $d = 4$, $\eta = 0.02$ (continuous line), $\eta = 0.05$ (dotted line) and $\eta = 0.08$ (dashed line).

Right panel: $q = 1$, $\eta = 0.05$, $d = 5$ (continuous line), $d = 8$ (dotted line), $d = 11$ (dashed line).

calculated values of deficit angle are almost independent of variation of nonlinearity parameter because its effect is so small. r_{ext} is an extremum in which for $r_{+1} \leq r_+ \leq r_{+ext}$, deficit angle decreases where r_+ increases while for $r_{+2} \geq r_+ \geq r_{+ext}$, deficit angle is an increasing function of r_+ (see Fig. 11). Finally for large values of r_+ ($r_{+2} \leq r_+$) similar behavior as for case of small values of r_+ ($0 < r_+ < r_{+1}$) is observed. Calculated values of deficit angle are almost independent of nonlinearity parameter and are almost the same. r_{+1} , r_{+2} , r_{+ext} and related deficit angle to this extremum are increasing functions of nonlinearity parameter. As for the effects of dimensions, it is evident from plotted graphs that r_{+ext} (and related deficit angle) is a decreasing (increasing) function of dimensions (see right panel in Fig. 10). These figures indicate that there exist regions, in which calculated values of deficit angle for different dimensions lead to almost same result and it is almost independent of dimensions.

Here, we present a geometric interpretation for negative deficit angle. Considering a two dimensional plain, we can cut segment of a certain angular size and then sew together the edges to obtain a conical surface. The deleted segment from the plan is known as deficit angle with positive values. Now, we imagine a new situation when a segment is added to a new plane to obtain a flat surface with a saddle-like cone (for more details one can see Fig. 2 in Ref. [45]).

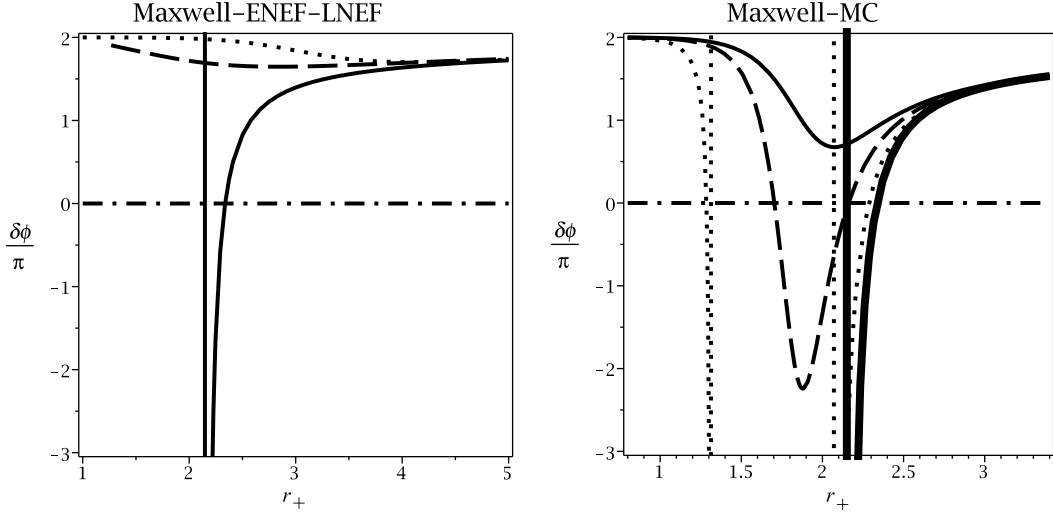


FIG. 11: $\delta\phi/\pi$ versus r_+ for $l = 1$, $q = 4$ and $d = 4$.

Left panel: Maxwell case (continuous line), ENEF case for $\beta = 5$ (dotted line) and LNEF case for $\beta = 5$ (dashed line), respectively.

Right panel: Maxwell case (bold line), MC case for $\eta = 0.02$ (dotted line), Maxwell Correction case for $\eta = 0.05$ (dashed line) and MC case for $\eta = 0.08$ (continuous line).

This added segment is corresponding to a negative deficit angle (or surplus angle) [45, 46]. We should mention that although the deleted segment is bounded by the value of 2π the added segment is unbounded. Therefore, we conclude that the range of deficit angles is from $-\infty$ to 2π .

V. CLOSING REMARKS

In this paper, we supposed that the geometry and matter field of spacetime come from the Lovelock gravity and NED. At first, we considered a suitable static metric to find horizonless magnetic solutions. We found that for having the real electromagnetic field, we should consider a lower bound (ρ_0) for the coordinate ρ . We discussed the geometric properties of EN, GB and TOL solutions and found that although these solutions have no curvature singularity, there is a conical singularity at $r = 0$ with a deficit angle $\delta\phi = 8\pi\mu$, where one can interpret μ as the mass per unit volume of the magnetic brane. In addition, we found that both the NED and the Lovelock gravity do not affect the asymptotically behavior of the solutions, and in other words, obtained solutions are asymptotically AdS. We obtained deficit angle of the conical geometry and investigated the effects of Lovelock gravity and NED. At first, calculated values for deficit angle showed that it is independent of the GB and the TOL parameters. In other words, we found that the Lovelock parameters do not affect the deficit angle. This result comes from the fact that the value of second derivatives of metric function does not depend on the Lovelock coefficients which is the consequence of geometric properties of $t = \text{constant}$ and $r = \text{constant}$ hypersurface (this hypersurface is a Ricci flat manifold). This behavior is similar to property of Ricci-flat black holes in higher orders of the Lovelock gravity, in which their horizons and conserved quantities of black hole do not depend on the Lovelock parameters.

We also investigated the effects of nonlinear electrodynamics. Although both ENEF and LNEF branches are BI type, they have different nature. We found that there is a minimum value for the nonlinearity parameter where for $\beta \leq \beta_{\min}$ the deficit angle was not real. This is because of the behavior of Lambert function which is present in ENEF branch and the logarithmic function which is appeared in LNEF branch. We also showed that considering higher dimensional solutions, β_{\min} may be changed and for certain dimensions, the deficit angle is real for arbitrary β ($\beta_{\min} < 0$). We found that the deficit angle is an increasing function of nonlinearity parameter in ENEF whereas for LNEF it showed opposite behavior. We also saw that increasing the charge parameter leads to increasing β_{\min} while for increasing r_+ , the value of β_{\min} decreased.

Looking at the behavior of deficit angle versus r_+ , we found that there is an $r_{+\min}$ where for $r_+ \geq r_{+\min}$, the deficit angle is real valued. Moreover, we found that for small values of the nonlinearity parameter, the deficit angle is only an increasing function of r_+ whereas for increasing value of β there will be r_{+ext} in which for $r_{+\min} \leq r_+ \leq r_{+ext}$ the deficit angle is a decreasing function of r_+ and for $r_+ \geq r_{+ext}$ it increases as r_+ increases.

Next step was devoted to introduce spinning magnetic branes which is horizonless. We found that for rotating magnetic branes there is an electric field in addition to the magnetic one. We employed the Gauss law and the counterterm method to calculate the electric charge, finite mass and angular momentum of rotating magnetic brane solutions. We found that the electric charge is proportional to the rotation parameters and it vanishes for the static solutions ($\Xi = 1$). We should note that vanishing the electric charge for $\Xi = 1$ is due the fact that the electric field, F_{tr} , vanishes for the static solutions.

As one can see for the weak nonlinearity power, the obtained deficit angle for different theories of nonlinearity has different values comparing to the Maxwell theory. One may argue that, for large values of β , the obtained values for deficit angle, should lead to those of the Maxwell theory and support this statement with fact that for large values of β , these two electromagnetic fields become Maxwell theory. This idea is an acceptable one, when we are only dealing with the electromagnetic fields. But in calculation of deficit angle, we are using the second derivation of metric function. Due to different structures of nonlinear theories (logarithmic and exponential ones), it is most likely that this property of these two nonlinear electrodynamics (for large values of β , they lead to the Maxwell theory) is not preserved and therefore, the obtained values are different. In other words, one may expect to see different values for deficit angle even for large values of nonlinearity parameter and they are not necessarily the same as Maxwell ones. It means that, although these two types of nonlinear theory are BI type and for $\beta \rightarrow \infty$ they lead to same result, they are completely different theories with their different characteristics and properties.

In addition we found that plotted graph for the Maxwell theory, presents a divergency which is due to root(s) of f'' . While for considering nonlinear theories, the divergency vanishes and calculated values of deficit angle and plotted graphs showed no divergency. In other words, in process of going from linear theory (Maxwell) to a nonlinear theory (logarithmic form or exponential one), calculated values of deficit angle will be divergence free and it has smooth behavior. Therefore, considering nonlinear theories, changes properties of solutions and solve the problems regarding the linear theory which is of the primary motivation of considering nonlinear electrodynamics. It is notable, that considering nonlinear theories put some restriction on values. In other words, there is a region in which the calculated values of deficit angle are not real. But this region is not where the divergency of the Maxwell theory exists. In other words, by considering suitable value of nonlinearity parameter, one can cover regions in which the Maxwell theory has divergency.

Later, we investigated the effects of nonlinearity as a correction. We found that this theory despite other two nonlinear theories (logarithmic and exponential ones) is always real value and there is no region in which deficit angle is imaginary. Plotted graphs of this theory also showed that, variation of nonlinearity parameter is only effective in a region ($r_{+1} \leq r_+ \leq r_{+2}$) and in other regions ($r_{+1} \geq r_+, r_+ \geq r_{+2}$), it is almost independence of this variation. Same behavior was seen for the effects of dimensions as well. It was shown that, the construction of this theory is in a way that for small and large values of r_+ the effect of nonlinearity part, decreases rapidly and almost vanishes and the structure of magnetic branes (cone-like) is similar to the Maxwell theory and it is as if there is no correction part. On the other hand for small values of correction parameter, not only it did not solve divergency of the Maxwell field, but it also added another divergency to it. In other words, two divergencies in the case of very weak correction parameter were seen in MC theory. This shows the fact that, this theory of nonlinearity and its deficit angle are quite sensitive to modification of correction parameter. This sensitivity is stronger even for small values of correction parameter. Although for some regions the calculated values of deficit angle are almost same as the one for the Maxwell theory, there is an effective range in which nonlinearity (correction) will be dominant and has the most contribution in deficit angle.

Other interesting issues in the results were existence of the negative, root and divergencies for deficit angle. Positive deficit angle is representing a cone like structure for the object. Whereas the negative deficit angle is denoted as extra angle which is know as surplus angle [45, 46]. This extra angle changes the shape of the object into a saddle like cone.

Finally it is worthwhile to think about the physical properties deficit angle as well as surplus one. In addition, One may investigate the possible wormhole solutions [47] of the mentioned models, This works are under examination.

Acknowledgments

We would like to thank the anonymous referees for valuable suggestions. We thank Shiraz University Research Council. This work has been supported financially by the Research Institute for Astronomy and Astrophysics of Maragha, Iran.

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